Mathematical Modeling of Pendulum Hand Pump

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***Abstract*—This paper presents the theoretical analysis of the dynamics of pendulum hand pump. The pump is the two stage mechanical oscillator whereby work done is obtained in the form of pumping. The reciprocating plunger pump is used which is basic conventional hand pump. The fundamental area of our research implies that a finite input in the form of angular displacement to be transmitted via a lever to the piston rod assembly that create sufficient force for pumping. The motion of whole system is governed by differential equations of engineering dynamics and fluid mechanics.**

***Keywords*—Pendulum; frequency; damping; nonlinear; kinematics;**

I. INTRODUCTION

The water is an essential commodity nature has blessed to the humans to suffice their sustenance. Not everyone has the direct access to it. In urban areas water is pumped via some sources such as electric motors to the folks but subjected only to the availability of water in that region. Pakistan has been blessed with multiple resources e.g. dams, rivers and unprecedented network of canals and on the top of it the freshwater channels that are the easiest way to pump it on the surface. On the contrary, the barren areas are deprived of direct supply of water from any source. They do possess water channels that requires huge human effort at the cost of low output. Such a dire need emphasizes to model and fabricate a design that can cater higher efficiency at relatively low losses.

Statistical studies depict that conventional hand pumps are used in underprivileged communities and offers extensive problems. Their ordinary design require massive inputs to operate which poses extraneous fatigue to the senior citizens, children and women. A substantial loss occur during this drive of motion. It includes greater damping losses, lower moments despite of huge applied forces, etc. Pendulum motion can supply huge amount of energy by just slight pushes.

The simulation carried the solutions of mathematical models that were framed by adding up all the forces to get the exact differential equations. The differential equations assist in comprehensive analysis of the whole system. Pendulum analysis involves the angular motion; whereas the lever input gets reduced due to two sources; one is provided by spring and other is due to viscous damping by the connecting rod which is assembled with the piston. Piston force causes pressure in the cylinder where fluid does the negative work and comes out of the spout. The net ultimate goal is to analyze the power requirements for pumping water at different heads. Modelling also assists to provide the pump components appropriate dimensioning and coefficients that help reduce the power losses, human efforts and make it more effective than other conventional pumps.

II. CAD MODEL OF THE SYSTEM

The proposed system consists of a pendulum, a lever mechanism, a push rod, and a piston [3]. The CAD model of the pendulum hand pump is shown in Fig. 1. This CAD model has been generated in Pro Engineering software. Fig. 1 shows the front view and Fig. 2 shows the isometric view of the proposed system. The angular motion of the pendulum is transmitted into the to-and-fro motion of the piston via the lever and push rod.

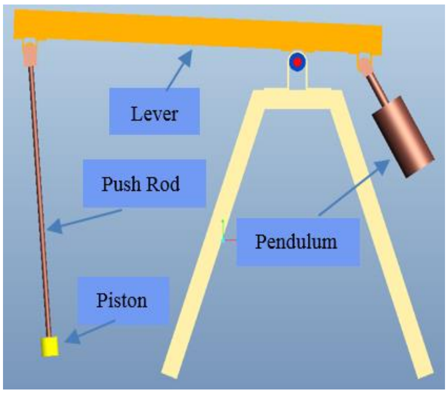

Fig. 1. Front view of the Pendulum Hand Pump

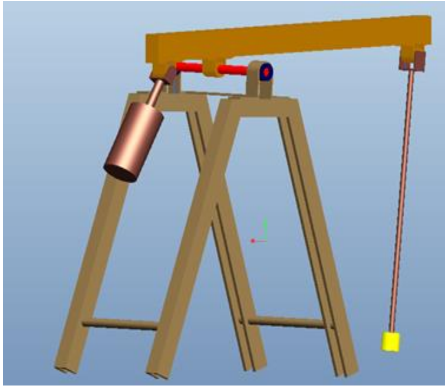

Fig. 2. 3-D Model of the Pendulum Hand Pump

Fig. 3. Pictorial view of the Pendulum Hand Pump

Fig. 4. Block diagram of the System

Fig. 3 shows the pictorial view and Fig. 4 shows the block diagram of the system [6].

III. MATHEMATICAL MODEL OF THE SYSTEM

Since power analysis involves the dynamics of the system. Motion is initiated by the forces so the models are framed by summing up all the forces on each individual components. The principal equation used throughout is:

$$F = m\ddot{x} \tag{1}$$

IV. A NONLINEAR PENDULUM MODEL

For the initial excitation of the system, a pendulum model is employed which consists of a bob having mass *m* attached to a thin rod of length *L*. It is assumed that the entire mass of the pendulum is concentrated in the bob. The angle θ is the angular displacement of the bob from its equilibrium position. The equation of motion for this pendulum is [7]:

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0 \tag{2}$$

It is clear that the equation of motion of the pendulum model (2) represents a second order system and the homogenous systems i.e. there is no time-dependent excitation force applied to the system. Since the proposed system's overall dynamic response depends upon the input excitation force, therefore, a harmonic excitation force $f(t)$ is considered for this system. Now (2) can be written as

$$\ddot{\theta} + \frac{g}{L}\sin\theta = f(t) \tag{3}$$

where ω_n^2 = g/L and is called the natural frequency of the system. The excitation force $f(t)$ is considered as a harmonic force and is defined as

$$f(t) = f_o \cos\omega t \tag{4}$$

where ω = excitation frequency

f_o = forcing excitation amplitude

Initiating the momentum of pendulum, we disturb its equilibrium:

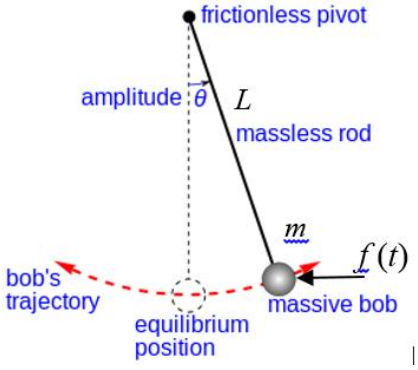

Fig. 5. Pendulum Model

It is almost not possible to find out the analytical solution of (3), therefore, it can be conveniently solved using numerical method, in this case Runge-Kutta method [7, 8], with the help of MATLAB programming language [7].

Isolating the highest order derivative of θ yields:

$$\ddot{\theta} = f(t) - \frac{g}{L}\sin\theta \tag{5}$$

The numerical integration of (5) yields the values of velocity. This velocity gradient sets the motion in the lever.

V. LEVER AND PISTON MODEL

The output from the pendulum model is the angular displacement (θ) which in turn is the input for the lever model.

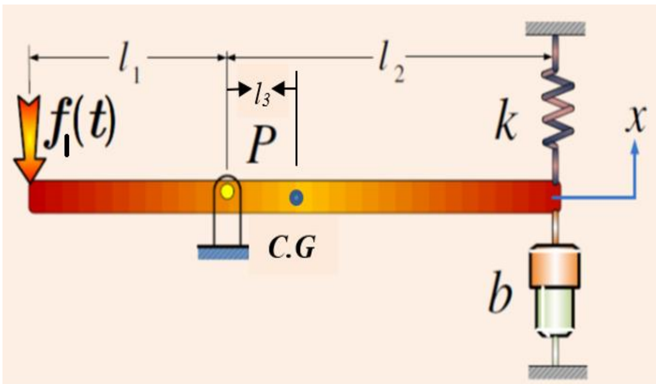

Fig. 6. Lever and Piston Model in Equilibrium

It is assumed that the θ is also exerted harmonically to the lever model. Fig. 6 shows the lever and piston model in equilibrium position [2]. The lever is pivoted at point P and it rotates about point P. The force $f_1(t)$, input force from pendulum model, is applied at the one end of the lever and at the other end the lever is allowed to displace by a linear distance x. A spring element, having stiffness k, is mounted at the other end. The piston model is represented by the damping element having the viscous damping constant b. It is assumed that lever and piston are displaced by the same displacement x. The total mass of the system is acting at the center of gravity ($C.G$) of the lever model and the reaction forces at point P are neglected.

Fig. 6 shows the free body diagram of the lever and pendulum model. The equation of motion for this model can be obtained by applying the Newton's second law of motion [4, 5]. The overall moment about the point P gives

$$m\ddot{x}l_3 + b\dot{x}l_2 + kxl_2 = f_1(t)l_1 \tag{6}$$

Assuming $l_3 = 1/3\ l_2$ and rearranging (6) for the highest order derivative of x, we get

$$\ddot{x} = \frac{3}{m}\left(f_1(t)l_1/l_2 - b\dot{x} - kx\right) \tag{7}$$

where, $f_1(t) = \sin(\theta)$ and θ is the angular displacement of the pendulum model. The numerical integrations of (7) yields the piston displacement x.

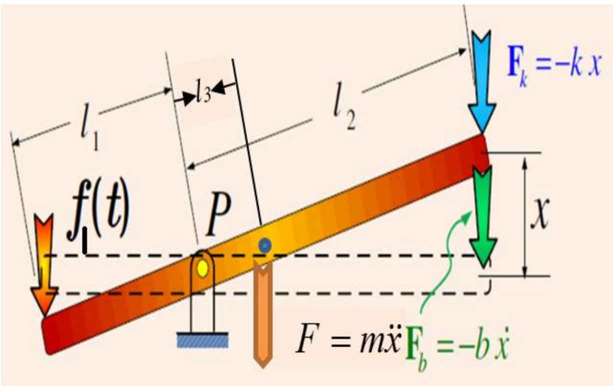

Fig. 7. Free Body Diagram of Lever and Piston Model

TABLE I. Pendulum Hand Pump Parameters

Pendulum Hand Pump Parameters		
Length of pendulum rod	L	1 m
Frequency of excitation force for pendulum	ω	5 rad/s
Amplitude of excitation force for pendulum	f_o	3 m
Distance from $f_1(t)$ and point P	l_1	0.4 m
Distance from k and point P	l_2	0.6 m
Distance from $C.G$ and point P	l_3	1/3 l_2
Spring constant	k	5 N/m
Viscous damping constant	b	1 N-s/m
Mass of the lever and piston model	m	4 kg

The important physical parameters of the pendulum hand pump model are listed in Table 1.

VI. Results And Discussion

This section presents the simulation analysis of the pendulum hand pump where the excitation frequency (ω) for the pendulum model is greater than the natural frequency (ω_n) of the pendulum model. It is important to mention here that this research study only focuses on the basic kinematics of the moving parts with the aim to investigate the response of the overall system under the influence of various excitation frequencies and initial conditions. The main objective of this simulation is to analyze the effects of different excitation force inputs on the dynamic response of the pendulum hand pump for the parameters given in Table 1. The total simulation time is 40 s.

Fig. 7 shows the simulation results of the proposed system where the initial condition for the pendulum is $\theta(0) = 0.5\pi$ radians and the natural frequency (ω_n) of the pendulum model is 3.1321 rad/s. The excitation frequency (ω) for the pendulum model is 5 rad/s which is greater than the natural frequency (ω_n) of the pendulum model. Fig. 7(a) shows the pendulum response against time for the excitation force input, whereas, Fig 7(b) shows the piston's linear displacement. It can be seen from Fig. 7(b) that for the given initial condition for pendulum and applied excitation force the total piston displacement is 0.38 m or 38 cm which is quite satisfactory for an ordinary hand pump.

The simulation results of Fig. 7(a) and Fig. 7(b) have been compared with the result of A. Akshaj [3]. The comparison of the results show that the results of proposed model are more realistic than the A. Akshaj [3] because they analyze the effects of excitation force on the response of the system. This helps in tuning the excitation frequency to certain range in order to get stable results. The analysis for the different excitation frequency is carried out below for deeper understanding of the relation of the input and output of the system which was not addressed in [3].

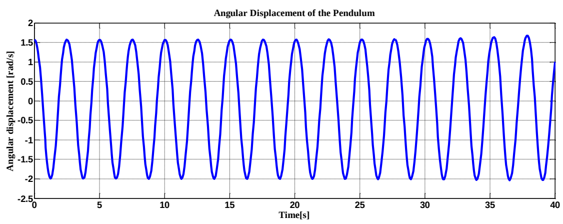

Fig. 7(a). Angular displacement of the Pendulum

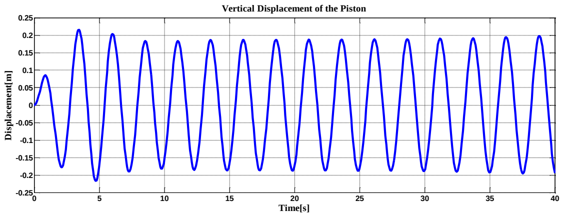

Fig. 7(b). Linear displacement of the Piston

The proposed pendulum hand pump model is further analyzed for a smaller excitation frequency (ω), i.e. the excitation frequency (ω) for the pendulum model is smaller than the natural frequency (ω_n) of the pendulum model. The excitation frequency (ω) in this case is 3 rad/s and the natural frequency is (ω_n) is same as before, i.e. 3.1321 rad/s. The remaining parameters of the system are same as in Table I.

It is evident from Fig. 8 that due to smaller excitation frequency (ω) than the natural frequency (ω_n), the response of the pendulum model and piston model is quite irregular. This shows that how the excitation frequency significantly influences the overall response of the pendulum. Even within 40 s of simulation time, the pendulum is not able to achieve any steady-state level, Fig. 8(a). This has the corresponding effect on the piston response, Fig. 8(b). This implies that for a stable response of the overall system, the excitation frequency (ω) for the pendulum model should be greater than the natural frequency (ω_n) of the pendulum model. It should be noted that greater difference of ω and ω_n will lead to reduced piston displacement. Therefore, the frequencies' difference should be kept as small as possible for the stable response of the system.

It was also observed that excitation frequency (ω) smaller than 5 rad/s increases the piston response but it has adverse effect on the pendulum response. However, if the excitation frequency (ω) is greater than 5 rad/s then the pendulum response is improved but the piston displacement is reduced. Therefore, it can be concluded that the excitation frequency for this proposed physical system should be kept closer to 5 rad/s which can also be termed as the critical frequency of this system.

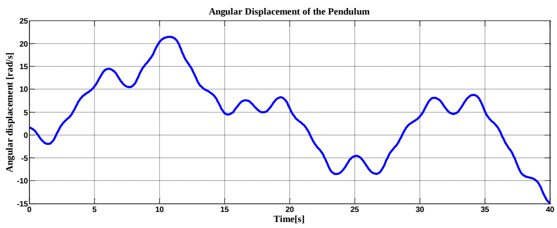

Fig. 8(a). Angular displacement of the Pendulum

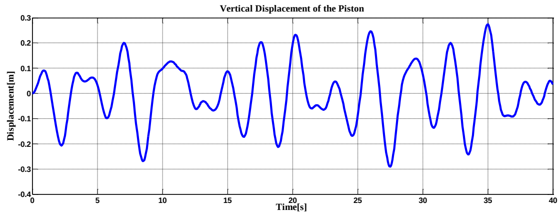

Fig. 8(b). Linear displacement of the Piston

VII. CONCLUSION

In this study a nonlinear pendulum model is used to power the lever and piston model for the applied excitation force to the pendulum. A complete kinematic approach is used in this study to analyze the dynamic response of the system. It was observed that a smaller excitation frequency (ω) than the natural frequency (ω_n) of the pendulum model leads to an unstable response of the system, whereas, greater excitation frequency (ω) than natural frequency (ω_n) of the pendulum model gives the satisfactory results.

I. FUTURE WORK

This paper would be the roadway to the numerical solution of kinetic aspects of motion of single acting reciprocating pumps. It will ease the complex numerical study of power and efficiency of the pump. Having set up the motion in components, water pump can be facilitated according to the desired values. An input frequency control and its variation can prove to yield different pressure heads. This study requires crucial and in-depth study of fluid mechanics as well.

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REFERENCES

[1] R.C. Hibbler, Engineering Dynamics, "Vibrations", Thirteenth Edition.

[2] A. Azeem Bazoune, System Dynamics and Control, "Mechanical Elements".

[3] A. Akshaj, "Design and Fabrication of Hand Water Pump with a Pendulum", ISRASE First International Conference on Recent Advances in Science & Engineering-2014 (ICRASE-2014).

[4] D. A. Stephen, "Numerical Simulation of Reciprocating Pumps and Air Operated Motors", Fluids/Solids Handling, August 2002.

[5] V. S. Sorokin, "Dynamics of a Nonlinear Mechanical System Containing Pendulums", EUROMECH Colloquium 503, Department of Mechanics and Control Processess, St. Petersburg State Polytechnical University, St. Petersburg, Russia, Sept 27-Oct 2, 2009.

[6] E. R. S. Carlos, "Development of an Extensible Interactive Modeling, Simulation, Animation, and Real-Time Contro (MoSART) Cart-Pendulum-Seesaw Environment: A Tool for Enhancing Research and Education", A Thesis Presented in Partial Fulfillment of the Requirements for the Master of Science Degree, Arizona State University, December 1999.

[7] W. J. Palm III, "Introduction to MATLAB for Engineers", 3rd Edition, McGraw Hill, New York, 2011.

[8] S. C. Chapra and R. P. Canale, "Numerical Methods for Engineers", 6th Edition, McGraw Hill, United States of America, 2009.